Does Cold Fusion Exist and is it Measurable?

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The dynamical nature of the cold fusion problem is considered. A general model of two slow charged particles interacting in a metal crystal is developed. It is shown that there exists a resonance state of this system. The value of the positive real resonance energy depends on the properties of the metal crystal. An estimate of the probability of cold nuclear fusion in condensed matter is given.

Recently, the d + d nuclear fusion in Pd-D (i.e., Pd metal crystal with absorbed deuterium atoms) at room temperature has received much attention. Various experimental papers were published in which this effect was studied, however with contradictory results [1–3].

From the theoretical point of view, cold nuclear fusion was considered in [4–6]. It was shown in [4] that this process possibly may take place without production of neutrons, if one considers the Pd-D crystal as a cold plasma. In [5], estimates of the screening effects of the electrons on the nuclear Coulomb barrier were given, and in [6], a calculation based on the effective potential of two interacting deuterons in a palladium lattice was presented.

In the present paper, the dynamical nature of the cold fusion is considered. Attempting to clarify this problem, we have developed a general model of two slow charged quantum particles interacting in a metal crystal. We have proven that there exists a resonance state of this system and that the value of the positive real resonance energy depends on the properties of the metal crystal. So we probably have found a dynamical pairing of the positively charged particles in a metal crystal. This result made it possible to estimate the probability of cold nuclear fusion in condensed matter.

In developing this general model, we made some special assumptions, since the system under consideration has special features due to the type of the interaction potential of two slow positively charged particles in a metal crystal, i.e., in the electron gas of the metal. Taking one particle with the charge Z_1 as a center fixed in the electron cloud of density $\varrho(r)$, where $r(x, y, z)$ is the distance from the center, we treat now the problem as a scattering of the other incoming particle (with the charge Z_2) in a field obtained as the superposition of the two potential fields. One potential, of the Coulomb point central field, is given by

$$V_c(r) = V_{oc}(r_0/r); \quad V_{oc} = \left[\frac{e^2}{4\pi\varepsilon_0}\right] \cdot \frac{Z_1 \cdot Z_2}{r_0}, \quad (1)$$

and the second potential, which is due to the charge of the electron cloud with density ϱ_0, is given by the potential function

$$V_{el}(r) = -V_{0\,el}(r_2/r_0^2), \quad (2)$$

where

$$V_{0\,el} = \left[\frac{e^2}{4\pi\varepsilon_0}\right] \frac{2\pi}{3} Z_2\,\varrho_0\,r_0^2\,,$$

$$\varrho_0 = \frac{3}{4\pi} r_s^{-3}$$

and r_s is the Wigner-Seitz radius for an electron in atomic units ($r_0 = 0.5 \cdot 10^{-8}$ cm), e the electron charge and ε_0 the dielectric constant. It is easy to see that, whereas the Coulomb central potential is decreasing with the distance as r^{-1}, the potential of the electron cloud increases with growing distances as r^2. Because of this fact, the interaction between the two charged particles takes place effectively only for a distance smaller than $R_0 \approx r_s$, which can be found from the relation $V_c(R_0) = -V_{el}(R_0)$.

To describe the interaction of two deuterons, we analyze the scattering parameter $k\alpha$, where k is a relative wave vector of two particles and α is a distance smaller than R_0, which for the given problem is of the order of a few r_0. E.g., for the slow particles with reduced mass μ, corresponding to a system of two deuterons, and with the energy $E \geq 0.02$ eV, the norm of the wave-vector satisfies the inequality $k > 5 \cdot 10^8\ \text{cm}^{-1}$; therefore, the scattering parameter $k\alpha$ of the incoming par-

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ticle appears to be larger than one. Thus it is possible to treat this problem in the quasi-classical approximation. Hence, to analyze the resonances of the given two particle system, we may consider the solution of the Lippman-Schwinger equation in the "Eikonal approximation" [7].

As is well known, the integral equation for the wave function of a particle with the energy $E = k^2/2\mu$, scattered by the potential $V(r) = -[V_c(r) + V_{el} r)]$, has the form

$$\Psi_K^+(\boldsymbol{r}) = \frac{1}{(2\pi)^{3/2}} \exp(i k z) + \int d^3 r' \, G_0^+(|\boldsymbol{r} - \boldsymbol{r}'|) \cdot V(\boldsymbol{r}') \cdot \Psi_K^+(\boldsymbol{r}'), \quad (3)$$

where $G_0^+(|\boldsymbol{r} - \boldsymbol{r}'|)$ is the Green function, defined as

$$G_0^+(|\boldsymbol{r} - \boldsymbol{r}'|) = (2\pi)^{-3} \int d^3 \boldsymbol{p} \, \frac{e^{i \boldsymbol{p}(\boldsymbol{r} - \boldsymbol{r}')}}{k^2 - p^2 + i\varepsilon}. \quad (4)$$

Here the direction of $\boldsymbol{k}$ was chosen to coincide with the z-axis. Let us define the function $\Phi_K(\boldsymbol{r})$, which is connected with $\Psi_K^+(\boldsymbol{r})$ by the relation

$$\Phi_K(\boldsymbol{r}) = (2\pi)^{3/2} \cdot e^{-ikz} \cdot \Psi_K^+(\boldsymbol{r}) \quad (5)$$

and satisfies the equation

$$\Phi_K(x, y, z) = 1 + (2\pi)^{-3} \int d^3 \boldsymbol{p} \, d^3 \boldsymbol{r}'' \frac{e^{i(\boldsymbol{p} - \boldsymbol{k})\boldsymbol{r}''}}{k^2 - p^2 + i\varepsilon} \cdot V(|\boldsymbol{r} - \boldsymbol{r}''|) \, \Phi_K(\boldsymbol{r} - \boldsymbol{r}''). \quad (6)$$

In the Eikonal approximation, we have to take the vector $\boldsymbol{p}$ almost equal to the vector $\boldsymbol{k}$ (in value and in direction). Thus (6) transforms to

$$\Phi_K(x, y, z) = \left[1 + \frac{i}{2k} \int_{-\infty}^{z} dz' \cdot V(x, y, z') \, \Phi_K(x, y, z') \right] \quad (7)$$

which has a formal solution of the type

$$\Phi_K(x, y, z) = \exp\left\{ -\frac{i}{2k} \int_{-\infty}^{z} V(x, y, z') \, dz' \right\}. \quad (8)$$

One can see that in the Eikonal approximation the wave function $\Psi_K^+(\boldsymbol{r})$ has the form

$$\Psi_K^+(\boldsymbol{r}) = (2\pi)^{-3/2} \cdot e^{ikz} F_1(x, y, z) \, F_2(x, y, z), \quad (9)$$

where the factors

$$F_1(x, y, z) = \exp\left\{ -\frac{i}{2k} \int_{-\infty}^{z} -V_c(x, y, z') \, dz' \right\}$$

and

$$F_2(x, y, z) = \exp\left\{ -\frac{i}{2k} \int_{-\infty}^{z} V_{el}(x, y, z') \, dz' \right\},$$

corresponding to the different potentials, are independent. Let us examine the asymptotic behavior of $\Phi_K(\boldsymbol{r})$ as $r \to \infty$. As the function $\Phi_K(\boldsymbol{r})$ evidently must be at least bounded and $V_{el}(r)$ increases asymptotically as r^2, analyzing (7) we may conclude that $\Phi_K(\boldsymbol{r})$ approaches zero faster than r^{-2} for $z \to \infty$. It follows that, for some value of k, the function $\Phi_K(\boldsymbol{r})$ is in $L^2(R^3)$ and the system of two particles has a bound state with a positive energy. This result is not quite surprising, since for the one-dimensional two-body problem corresponding to the potential (2), the point spectrum on the whole real energy axis exists [8, 9].

This bound state, embedded in the positive continuous spectrum, is not stable and becomes a resonance with a real positive energy close to E_0 by an arbitrarily small perturbation.

The wave function (8) of this state, obtained in the Eikonal approximation, corresponds to an effective potential in the ordinary Schrödinger equation which has such a form that a coherent reflection on the potential walls occurs leading to a stationary wave in a bounded region. (For a similar example cf. [9], Section XIII.13.)

To estimate a probable value of the positive bound state energy, we analyzed the singularities in the k-plane of the scattering amplitude for two particles interacting in the metal crystal. Here we considered the scattering amplitude obtained in the quasiclassical approximation. As is well known [7], this amplitude has the form

$$f(\Theta) = \frac{k}{2\pi i} \int d^2 \boldsymbol{b} \cdot \exp\{i \boldsymbol{K} \boldsymbol{b}\} \, [B_1(b, k) \cdot B_2(b, k) - 1], \quad (10)$$

where

$$B_1(b, k) = \exp\left\{ -\frac{i}{2k} \int_{-\infty}^{\infty} V_c(b, z) \, dz \right\}, \quad (11)$$

$$B_2(b, k) = \exp\left\{ -\frac{i}{2k} \int_{-\infty}^{\infty} V_{el}(b, z) \, dz \right\}. \quad (12)$$

The vector $\boldsymbol{K} = \boldsymbol{k} - \boldsymbol{k}'$ is the difference between the initial and final momentum and $\boldsymbol{b}$ is a two-dimensional projection of the radius vector $\boldsymbol{r}$ on the plane orthogonal to the z-axis.

In our special problem of cold fusion, the variables z and b are in the interval $[z_0, \alpha]$. As we treat here an isolated two-deuteron scattering in the Pd-D crystal, the value of α is half the mean distance between the Pd and D-ions in a cell. These values approximately correspond to $0.25\, r_s$ [6, 10].

We should point out that, in the Eikonal approximation, the potentials $V_c(r)$ and $V_{el}(r)$ give two independent contributions B_1 and B_2 to the scattering amplitude. Thus to analyze the singularities of $f(\Theta)$, we considered the properties of the factor B_2 only. For the case of a weak potential (2) in the interval $[r_0, \alpha]$, one can expand the exponent in (12) and, taking the first term of this expansion, calculate B_2:

$$B_2 = \left[1 + \frac{i}{k} \int_{z_0}^{\alpha} V_{el}(b, z)\, dz \right]^{-1}. \tag{13}$$

Thus calculating the integral in (10) by partial integration, we extracted the singularities from the expression for the scattering amplitude and obtained the largest positive bound state energy in the vicinity of the room temperature energy in the form $E_0 \simeq [A \cdot [\alpha/r_s]^6 \cdot 10^5]$ eV, where the value of the constant depends on the mean distance 2α of the deuterium atoms in the Pd-D crystal and lies in the interval $0.25 \div 0.5$.

The numerical estimates give the value of this energy as about $E \sim 1$ eV, and the mean distance, d, of the two interacting deuterons in the corresponding state is then of the order of 10^{-10} and also depends on the density of the electron cloud and on the mean distance of the absorbed deuterons in the Pd-D crystal, i.e., on r_s and 2α. This means that the presence of the electron cloud in the Pd-D crystal leads to a dynamical pairing of the two deuterons involved in the scattering process or, in other words, to the localization of the two-deuterons in a sufficiently small volume (their distance is about 200 times smaller than the equilibrium distance in the D_2 molecule). Thus the nuclear fusion may occur. The D-D distance in the "dynamical pair" in the Pd-D crystal is of the same order as the mean distance between two deuterons in muon catalyzed fusion reported in [11].

According to [6, 10] one can find ΔE_0 which arises from the uncertainty of α. This makes it possible to roughly estimate the upper limit of the resonance life time, which is of the order of 10^{-10} sec and therefore much larger than nuclear time.

In conclusion, we may summerize as follows: the application of our model to the problem of electrochemically induced cold fusion is based on a pair-interaction of the incoming deuteron with an absorbed deuteron in the Pd-electrode. It is clear that this assumption is an approximation which is meaningful as long as the influence of the other absorbed deuterons may be considered as a perturbation, which means that the mean distance, 2α, between two absorbed deuterons cannot be too small, in particular it clearly cannot be smaller than $2r_0$. In this case, the mean distance of the two interacting deuterons, d, in the resonance state will be of the same order 10^{-10}. The smallness of d (with respect to α) also justifies the application of our model to the problem of cold fusion. (The whole problem will, of course, also depend on temperature.)

The aim of the present paper certainly was not to obtain quantitative results; we rather wanted to clear qualitatively the following points:

(1) is electrochemically induced cold fusion possible, and

(2) if so, may it be considered as a weak or a strong effect.

To answer the second question, let us make a rough estimate of the upper limit of the number of neutrons emitted by the palladium electrode in the experiment described in [3]. If we assume that cold fusion is a surface effect which takes place in the first ten monolayers, then for the experimental conditions of [3], the upper limit of the number of emitted neutrons should be of the order of about $1-10$ particles per second, so that it should, indeed be a weak effect, in accord with the result of [3].

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